A stick of mass density per unit length p rests on a circle of radius R. The stick makes an angle φ with the horizontal and is tangent to the circle at its upper end. Friction exists at all points of contact, and assume that it is large enough to keep the system at rest. Find the friction force between the ground and the circle.

The stick has total weight is pgL. Let the ball have weight Mg. There exists a set of forces acting on the stick and the ball. Forces acting on the stick are :

1. $F_{g/s}$, frictional force between ground and stick. Directed rightwards to prevent the stick from slipping rectilinearly left due to normal force from ball.
2. $N_{g/s}$, normal force between ground and stick.
3. N, normal force between ball and stick, perpendicular to stick at point of contact with ball, as they lie with surfaces tangent.
4. F, frictional force between ball and stick, parallel to stick. This points upwards, as a rectilinear push of the ball to the right would shift the contact point lower.
5. pgL, weight of stick. Directed downwards from a point exactly halfway along the stick's length.

Forces acting on the ball are :

1. $N_{g/b}$, frictional force between ground and ball. Directed leftwards to prevent ball from slipping rectilinearly right due to normal force from stick.
2. $N_{g/b}$, normal force between ground and ball.
3. Mg, weight of ball. Directed downwards from geometric center of ball.
4. N, normal force between ball and stick.
5. F, frictional force between ball and stick. By Newton's third law, this is equal in magnitude but opposite in direction to the friction by the ball on the stick.

By force analysis on the stick in x and y components :

$$N\sin\varphi - F\cos\varphi = F_{g/s}$$
$$N\cos\varphi + F\sin\varphi + N_{g/s} = pgL$$

By force analysis on the ball in x and y components :

$$N\sin\varphi = F_{g/b} + F\cos\varphi$$
$$N_{g/s} = Mg + N\cos\varphi + \mathrm{F}\sin\varphi$$

By torque analysis on the stick :

$$\frac{pgL\cos\varphi}{2} = N$$

By torque analysis on the ball, using both the geometric center of the ball and its contact point with the ground as pivots :

$$F_{g/b} = F$$

$$\text{N}\sin\frac{\varphi}{2} = F\cos\frac{\varphi}{2}$$

The forces at any contact points with the ground would also involve the frictional coefficient between the surface, and the weight of the object downwards the contributes to its normal force. As the mass of the ball is not known, any equations involving m_{ball} and N_{ball} cannot be used – this eliminates y- staticity of forces on the ball.

From torque analysis of the ball :

$$F_{g/b} = F$$

is found, eliminating the need for $\mu_{g/b}$ and $N_{g/b}$ to be known. The equation from x-component force analysis of the ball can hence be used :

$$N\sin\varphi = F + F\cos\varphi$$

Knowing N from torque analysis of the stick, F can be solved for :

$$\frac{pgL\cos\varphi}{2}\sin\varphi = F(\cos\varphi + 1)$$

$$F = \frac{pgL\cos\varphi}{2(\cos\varphi + 1)}\sin\varphi$$

Observing the geometry of the ball, $\varphi/2$ subtends a right angle triangle, with the right angle at the contact point between ball and stick. tan $(\varphi/2)$ can be defined trigonometrically :

$$\tan\frac{\varphi}{2} = \frac{r}{L}$$

Using the identity trigonometric identity :

$$\frac{\sin\varphi}{\cos\varphi + 1} = \frac{2\sin\frac{\varphi}{2}\cos\frac{\varphi}{2}}{2\cos^2\frac{\varphi}{2} - 1 + 1}$$
$$= \tan\frac{\varphi}{2}$$

Substituting into the expression for F :

$$F = \frac{pgL\cos\varphi}{2}\frac{\sin\varphi}{\cos\varphi + 1}$$
$$= \frac{pgL\cos\varphi}{2}\tan\frac{\varphi}{2}$$
$$= \frac{pgr\cos\varphi}{2}$$

Which is the answer to be found.